Preliminary Model for Radiometric Design and Analysis of the Breakthrough Starshot Initiative

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ABSTRACT

A model has been developed for preliminary examination of radiometric properties pertaining to the Breakthrough Starshot Initiative. This project aims to send a nanocraft to the Proxima Centauri system. The nanocraft will deploy a solar sail driven by a 100 GW laser beam at 1064 nm projected from a segmented aperture of several kilometers in diameter[1]. The radiometric model uses both MATLAB and FRED to quantify and characterize the light returning from the sail while also predicting the nanocraft's kinematic motion. Photon counts and wavelengths along with craft position, velocity, and acceleration at any time throughout the duration of the launch are among the quantities modeled. Since the nanocraft is to be accelerated to 20% the speed of light, relativistic effects must be considered[2]. In particular, the Doppler shifted reflected drive light is of significance since it may be suitable for wave-front sensing once the shift is sufficient to separate it from the unshifted drive light at the projector. Additionally, estimates of the photon flux support predictions of the amount of light the system will be able to collect for wave-front sensing. All of these quantities are calculated using an iterative process which takes known initial conditions and integrates forward in time to provide the desired properties at every time interval. The model allows system parameters (e.g., reflectivity of the sail, BRDF profiles, projector fill factor, etc.) to be varied for optimization of the drive system design. The model will continuously be updated as advancements in both projector and sail designs are made. An expanded model including other phenomena, such as sail self-emission, transmission, and irradiance distribution patterns at the solar sails, will follow.

Keywords: Radiometry, Radiative transfer, Active or adaptive optics, Wave-front sensing, Relativity

1. INTRODUCTION

The Breakthrough Starshot Initiative intends to utilize photon drive light as a means of accelerating the nanocraft to 20% the speed of light over the course of 600 seconds[1]. In order to do this the drive light needs to be coupled to the nanocraft's solar sail as efficiently as possible, despite constant fluctuations in the Earth's atmosphere that will cause phase errors across the segments of the launch projector as well as piston shifts between them. Fast wave-front sensing with feedback to adaptive corrector elements in the projector optics will be used to minimize these atmospheric effects and maximize power incident on the sail[3]. A concept for the projector developed by Angel et al.[4] is shown in Figure 1.

In order to achieve accurate wave-front sensing a guide beacon will be needed. Within the first 30 seconds of launch an artificial beacon of some kind can be used and there are multiple methods for accomplishing this being examined[5]. However, after the first 30 seconds of launch the drive light reflected from the sail will be Doppler shifted such that it will be distinctly different in wavelength from the light emitted from the projector. Thus, theoretically the Doppler shifted light itself can be used as the beacon for the remaining 570 seconds of launch.

With this concept in mind, a model has been generated to quantify the light returning from the sail at any point over the duration of the launch. This will allow for a rigorous assessment of the suitability of Doppler shifted drive light as a guide beacon. The model presented can be broken down into two main components, the Kinematic Model and the Ray-Trace Model. Both models have their own architecture and outputs that when combined give a complete description of the return light for a large set of input parameters.

Parkin has conducted previous research into the general design of the greater Breakthrough Starshot system in his paper "The Breakthrough Starshot System Model"[1]. He does not do an in-depth radiometric analysis in his model but he does study the kinematic motion of the nanocraft and provides results that can be compared to some of those presented here.

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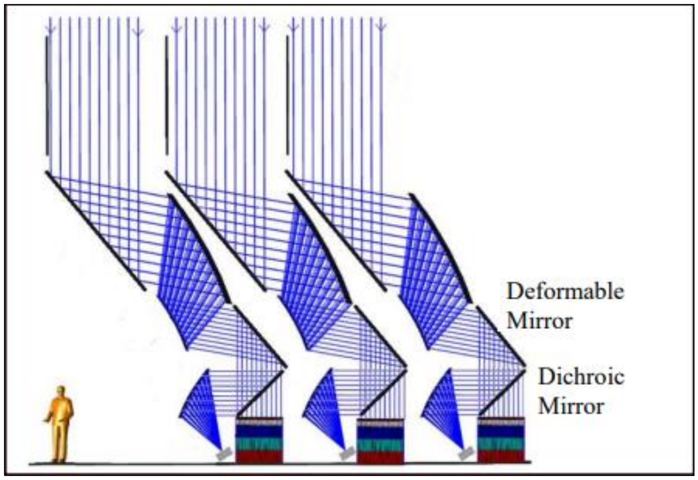

Figure 1. Concept Design of Launch Projector[4]

2. KINEMATIC MODELING

2.1 Model Architecture

The Kinematic Model characterizes the nanocraft's motion over the duration of the launch. This is done in MATLAB using an algorithm that loops over small, defined time steps. The primary output of the model is the craft's distance from the launch aperture at a given point in time. This distance value is then exported to the Ray-Trace Model for ray-tracing.

The distance value is found at each time-step through a series of calculations that take into account relativity and its associated effects. Several other parameters of interest (i.e., velocity, acceleration, force on sail, etc.) are calculated in the process. All of these values are recorded at given larger intervals. A flow chart of the iterative process is shown in Figure 2 while key input variables are identified in Table 1.

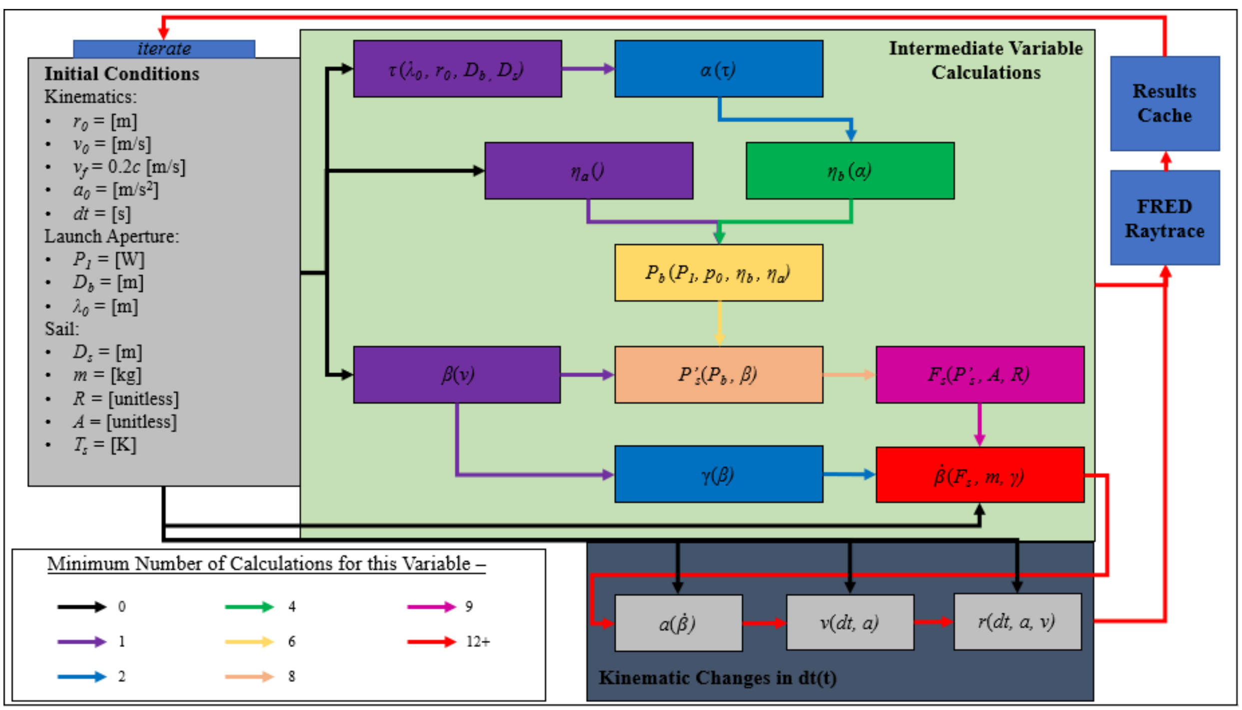

Figure 2. Kinematic Model Flow Chart

Table 1. Key Input Variables

Variable	Definition
r_o	Nanocraft's initial distance from projector
v_o	Nanocraft's initial velocity
v_f	Nanocraft's final velocity
a_o	Nanocraft's initial acceleration
dt	Time step in-between calculations
P_l	Power at projector
D_b	Projector diameter
λ_o	Wavelength of drive light at projector
D_s	Sail diameter
m	Nanocraft mass
R	Sail reflectivity
A	Sail absorptivity
T_s	Sail temperature

The series of calculations is illustrated by Parkin in his model[1] and begins with the given set of input parameters described in Table 1. Using the wavelength of drive light at the projector, the nanocraft's initial distance from the projector, and the projector and sail diameters the beam transfer efficiency (η_b) is calculated as given by Equations 1-6.

$$\tau = 2\pi \frac{\lambda_0 z}{\sqrt{A_s A_b}} = \frac{8\lambda z}{D_s D_b} \quad (1)$$

$$\alpha(\tau) = \sqrt{\frac{2\pi}{\tau}} \quad (2)$$

$$b = e^{\alpha^2} \quad (3)$$

$$\eta_1(a) = \frac{1}{4b^2}\left(\alpha^4 + \sqrt{\alpha^8 - 4\alpha^4 b + 4b^2 - 8b + 4}\right)^2 \quad (4)$$

$$\eta_2(a) = \left(\frac{\alpha^2}{2} - \frac{\alpha^6}{32} + \frac{7\alpha^{10}}{4608}\right)^2 \quad (5)$$

$$\eta_b = \begin{Bmatrix} \eta_1(a) & \text{if } \alpha > 1.21748051194181 \\ \eta_2(a) & otherwise \end{Bmatrix} \quad (6)$$

The beam transfer efficiency gives an indication of how well the beam is coupled to the sail as a fraction of the total light after propagation over the projector-to-sail distance. It is used in conjunction with the atmospheric transfer efficiency (η_a), which gives the fraction of total light transmitted through the atmosphere, to calculate the power successfully coupled to the sail (P_b) as given by Equation 7.

$$P_b = \eta_a \eta_b P_1 \quad (7)$$

The power coupled to the sail is then used alongside the relativistic velocity ratio (β) to calculate power at sail (P'_s) and force on sail (F_S) values adjusted for the relative speed between the projector and nanocraft. The relativistic Lorentz transform efficiency (γ) is also used alongside the adjusted force on sail value to give a final acceleration value scaled by the speed of light ($\dot{\beta}$). The sequence of calculations is given by Equations 8-12.

$$\beta = \frac{v}{c} \tag{8}$$

$$P_s' = \frac{1-\beta}{1+\beta} P_b \tag{9}$$

$$F_s = \frac{P_s'}{c}(A + 2R) \tag{10}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{11}$$

$$\dot{\beta} = \frac{P_b}{\gamma^3 m_0 c^2}\frac{1-\beta}{1+\beta}(A + 2R) \tag{12}$$

Thus, the nanocraft's new acceleration, velocity, and resulting distance from the projector after each time step are given by Equations 13-15.

$$a_0 = \dot{\beta} c \tag{13}$$

$$v_0 = v_{0i} + a_0(dt) \tag{14}$$

$$r_0 = r_{0i} + v_0(dt) + \frac{1}{2}a_0(dt)^2 \tag{15}$$

This final distance value can then by used in the Ray-Trace Model so that the radiometric analysis can be performed with the appropriate system geometry for the corresponding point in time. Additionally, the Kinematic Model indicates how other factors of the craft are changing in time. A summary of all calculated variables is provided in Table 2.

Table 2. Key Calculated Variables

Variable	**Definition**
τ	Unitless parameter
α	Unitless parameter
b	Unitless parameter
η_1	First solution to beam transfer efficiency
η_2	Second solution to beam transfer efficiency
η_b	Chosen solution to beam transfer efficiency
η_a	Atmospheric transfer efficiency
P_b	Power successfully coupled to sail
β	Relativistic velocity ratio
P'_s	Power at sail
F_s	Force on sail
γ	Lorentz transform efficiency
$\dot{\beta}$	Acceleration scaled by speed of light
a_0	Nanocraft's new acceleration
v_0	Nanocraft's new velocity
r_0	Nanocraft's new distance from projector

2.2 Model Results

The Kinematic Model was run for a given set of initial conditions and multiple output parameters were plotted as a function of time. The input parameters were drawn from a variety of sources and are all approximated theoretical values that can be easily adjusted as the system evolves. While several parameters, such as estimates of the projector and sail diameter, nanocraft mass, and power at the projector, were chosen based on estimates by Parkin and the Breakthrough Starshot committee[1], others, like the initial nanocraft distance and velocity, were chosen based on orbital calculations[5]. Additionally, some, such as the sail reflectivity and absorptivity, are placeholders until more accurate values are available. The input parameters are given in Table 3.

Table 3. Input Parameters

Variable	Value
r_o	$190x10^6$ [m]
v_o	$6x10^3$ [m/s]
v_f	$60x10^6$ [m/s]
a_o	0 [m/s^2]
dt	10^{-6} [s]
P_l	$35x10^9$ [W]
D_b	4000 [km]
λ_o	$1060x10^{-9}$ [m]
D_s	2 [m]
m	10^{-3} [kg]
R	0.999
A	0.001
T_s	100 [K]

Figure 3 shows plots illustrating how the main kinematic outputs (i.e., acceleration, velocity, and distance) as well as successfully coupled power, force applied to the sail, and the relativistic velocity ratio vary with time. In general, the results of the Kinematic Model with basic input parameters are roughly as expected. The acceleration decreases over the duration of the launch as a result of two main effects. Firstly, the power couples less and less successfully from the projector to the sail as time increases. This effect is most evident when considering the “knee” in both the acceleration and force plots. This discontinuity ultimately stems from the evolution of the beam transfer efficiency (η_b) with time. The analytical solution for this transfer efficiency can be derived from the fact that the beam eventually begins to spill off of the sail as a result of diffraction effects. This falling-off starts to become significant at around the 3 minute mark, which is where the discontinuity arises. Prior to this, the transfer efficiency is close to 1.0. Additionally, the redshift of the drive light leads to a decrease in acceleration. These two effects together reduce the acceleration of the nanocraft which in turn causes the velocity curve to level out as it approaches approximately 20% the speed of light.

2.3 Comparisons to Previous Research

As mentioned previously, the results of the kinematic model can be qualitatively compared to Parkin’s study, though quantitative comparison is difficult because of important differences between the input parameters used in the two cases. For example, the nanocraft mass and initial projector-to-nanocraft distance are 3.6 g and 60,000 km respectively in Parkin’s model[1] versus 1.0 g and 190,000 km used here. At this early stage in the project, the values of many key parameters have yet to be defined. Exploring the impact of various choices is the primary motivation for the present modeling work.

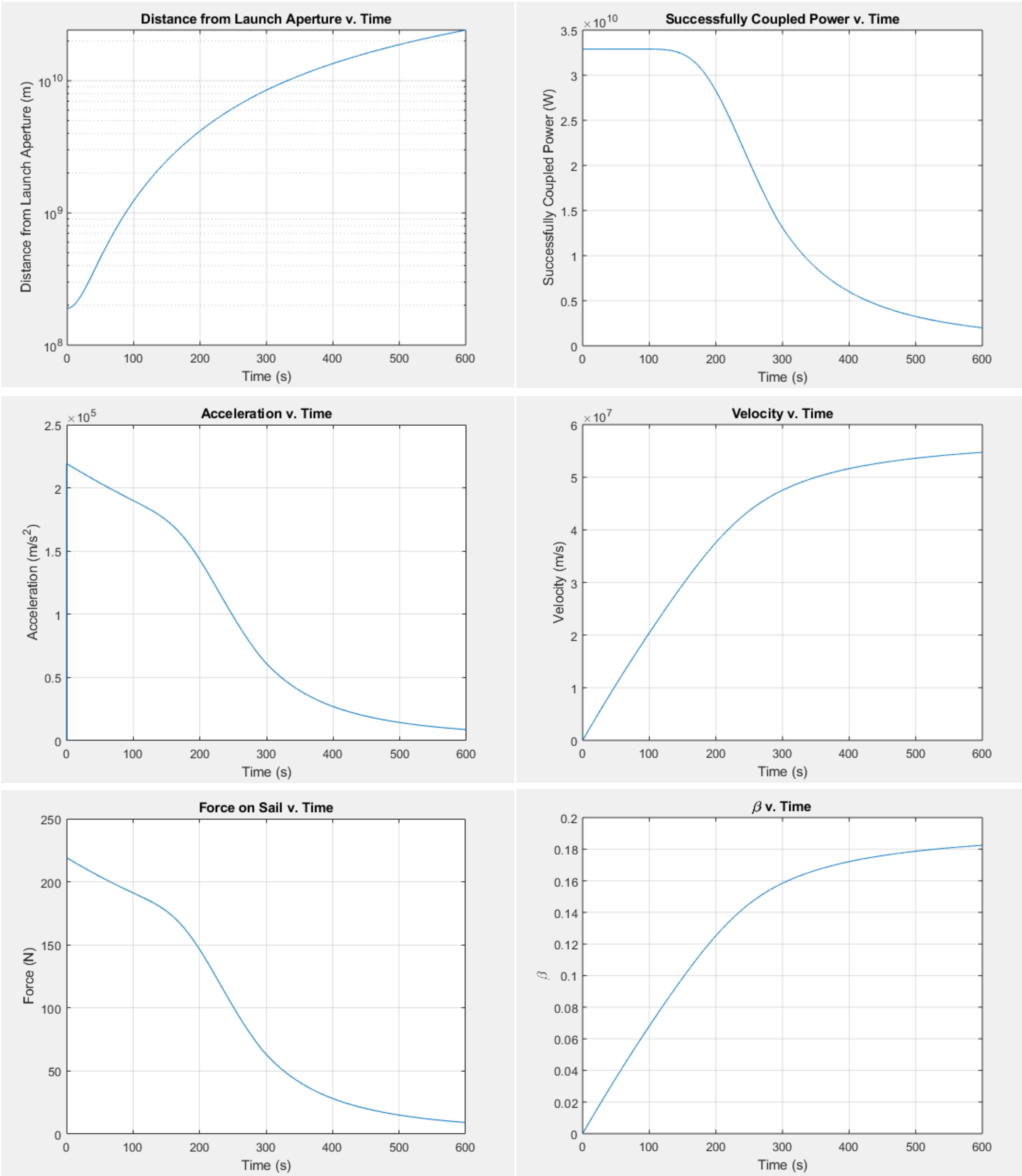

Figure 3. Key Kinematic Model Outputs.

The values of many output parameters are not identical to Parkin's, though they are similar in trend. Comparable plots for velocity and power are not given by Parkin, but acceleration, distance, force, and β are all provided. It is worth noting that in both of Parkin's acceleration and force plots the trends are similar to those shown in Figure 3 proceeding the 5 minute mark, but prior to that both quantities are relatively constant. This is due to the fact that Parkin assumes the launch aperture power will initially be throttled back to prevent the sail from overheating, and in turn, power dependent quantities are constant for the first portion of the launch[1]. It is worth noting this is due to assumptions about the properties of the sail material which have not yet been established by the Starshot project. Thermal effects have yet to be explored in the model presented here but will be considered moving forward.

3. RAY-TRACE MODELING

3.1 Model Architecture

The Ray-Trace Model characterizes the quantity and distribution of return drive light after reflection from the sail using non-sequential ray-tracing methods. The model is constructed so that values can be provided at any point in time larger than the chosen time-step (dt) used in the Kinematic Model. Thus, the model continues the iterative process of providing results based on input parameters, namely the distance values provided by the Kinematic Model.

The Ray-Trace Model is constructed in FRED and consists of relatively basic system geometry. The two main components of the model are the launch projector and sail, which are designed according to their respective dimensions. The two are separated by a distance r_o. FRED is ideal for this type of modeling as it is easy to adjust and alter a wide variety of parameters and specifications related to the system. It is worth noting that there are also atmospheric effects taking place right above the launch projector as a result of atmospheric turbulence. This can be modeled in FRED through a variety of techniques, such as changing the refractive index of the air slightly, adding particulates with specified reflectance and BRDF values, and allowing these factors to vary as a function of time. Additionally, the kinematic model already takes into account the impact this has on the nanocraft's motion through employment of the atmospheric transfer efficiency (η_a). Here only the most basic parameters are defined, as given in Table 3, but in future work these parameters, along with others, will be adjusted to optimize system performance. A simplified layout of the model's architecture along with a basic illustration of the geometry within FRED is displayed in Figure 4.

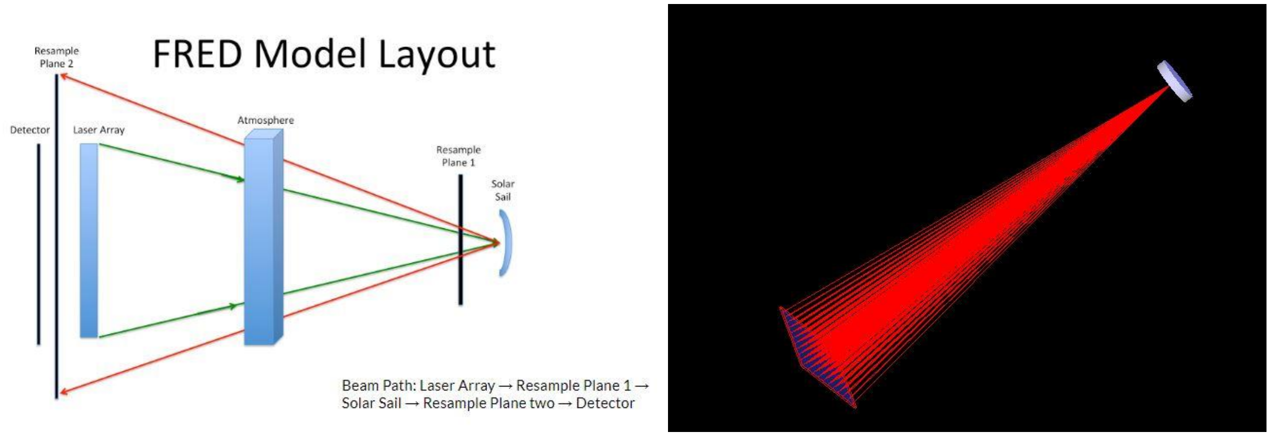

Figure 4. Ray-Trace Model Layout (left) and FRED Geometry (right).

Once the system is constructed within FRED, ray-traces can be performed to begin the characterization of the return drive light. A ray-trace is performed at the initial distance. Then, the model updates to a new distance between the projector and the nanocraft and the ray-trace is performed again. This procedure is carried out for the entire length of the launch at each r_o value, producing a complete profile of the beam. Using "analysis planes" within FRED the irradiance distributions both at the sail and back at the projector upon return can be examined at all points throughout the launch.

The extreme geometry of the system, a huge aperture and a propagation distance that varies over orders of magnitude, represents a modeling challenge. In particular, the transverse sampling of the beam must be adjusted at a number of points along the beam to prevent the problem from becoming numerically intractable. FRED allows this by positioning "resampling planes" at selected locations for each ray-trace and each r_o, as illustrated in Figure 4. Optimizing the location of these planes is the subject of on-going work. For that reason only the irradiance distributions at the starting distance of the nanocraft are provided below in Figure 5. However, once the resampling planes are positioned correctly the model will be used for all distance values.

3.2 Model Results

The irradiance distributions are provided for the starting r_o value of 190,000 km. The distributions shown in Figure 5 are given at points along the beam path 500 km and 10 mm prior to the sail's surface. The beam can be seen to come near focus at the 10 mm point. Here a Gaussian beam is modeled, although that will likely not be the final beam shape: a number of alternatives are under consideration that would stabilize the sail against lateral disturbances. To explore those, we will in future work take advantage of a feature within FRED to adjust the shape of the beam. However, for proof-of-concept purposes this illustrates the Ray-Trace Model's architecture and current capabilities.

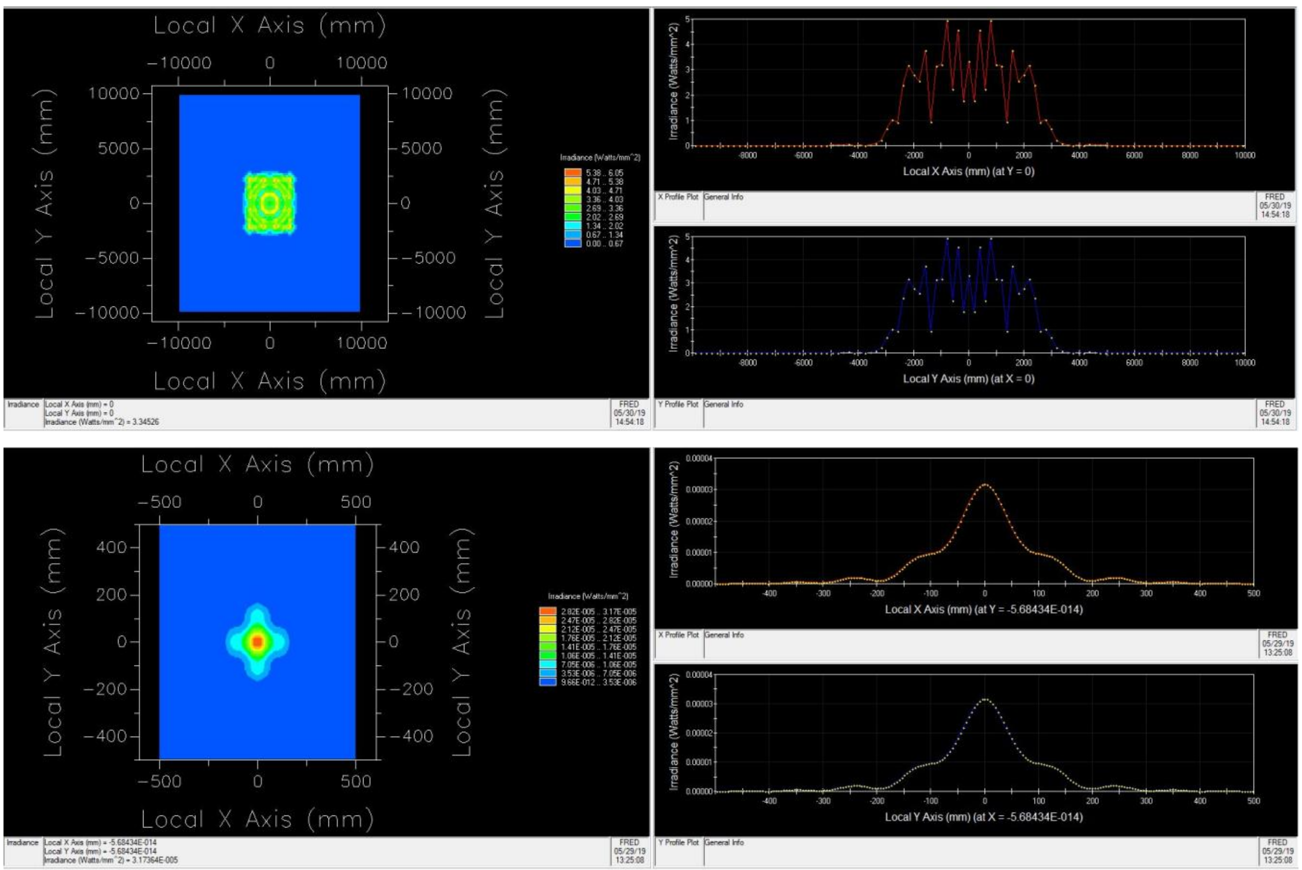

Figure 5. Irradiance profiles at 500 km (top) and 10 mm (bottom) from the sail.

Figure 5 shows a tightly focused beam with adaptive optics control to the diffraction limit. In the early stages of launch, this is not desirable because the small size of the beam compared to the sail would lead to extreme pressure gradients that would be induced across the sail. Instead, we envision that the beam would initially be somewhat defocused to fill the sail, with the magnitude of the focus term being reduced to zero as the angular subtense of the nanocraft shrinks below the diffraction limit of the launch projector. The Ray-Trace Model can readily accommodate such a varying focus.

4. SUMMARY AND FUTURE IMPROVEMENTS

A general model for predicting the radiometric performance of the reflected laser guide light has been established through the development of a segmented model including kinematic and ray-tracing portions in MATLAB and FRED respectively. The model described here is the preliminary version of a work in progress. The final model will refine the current physical properties and phenomena for an increasingly accurate simulation of the system. One of the primary areas of interest is the solar sail's design from both a geometric and materials standpoint. The model will be used to test the efficiency of different sail shapes (e.g. circular, spherical, conic, etc.) as well as various materials. The PSF shape of the beam is another area of interest that can be readily explored via this model.

In addition to large system variations like these, other parameters that will be explored in an effort to optimize the system performance include the effects of sail thermal self-emission, different reflectivity and bidirectional reflectance distribution functions (BRDF), various levels of absorption, potential changes to sail mass throughout flight, and induced polarization effects. FRED allows for easy customization and editing of many parameters, making this a strong starting point for the more robust model.

ACKOWLEDGMENTS

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REFERENCES

[1] Kevin Parkin, "The Breakthrough Starshot System Model," Parkin Research, San Francisco, July 2018.

[2] Neeraj Kulkarni, "Relativistic Spacecraft Propelled by Directed Energy," University of California Santa Barbara, October 2017.

[3] Michael Hart, "Wavefront Sensing for the Breakthrough Starshot Laser Launch Projector," College of Optical Sciences, University of Arizona, November 2017.

[4] Roger Angel, "Adaptive Optics for Starshot Beam Launch," Proc. AO4ELT6, (2019)

[5] Matthew Noyes, "Analyzing the Viability of Satellite Laser Guide Stars for Breakthrough Starshot," Proc. AO4ELT6, (2019).