Analyzing the Viability of Satellite Laser Guide Stars for Breakthrough Starshot

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ABSTRACT

The Breakthrough Starshot team plans to launch ultra-lightweight nanocraft to Proxima Centauri at 20% the speed of light with the propulsion of a high-powered ground based laser at 1064 nm. The proposed laser projector's aperture spans several kilometers and will require an adaptive optics (AO) pre-correction to properly focus through Earth's turbulent atmosphere. To measure the turbulence induced aberration above the projector system we have suggested that the Doppler shifted return light from the nanocraft be used as a beacon. For this method to work, the return light must be separable from outgoing laser light that is Rayleigh scattered off the atmosphere. However, the craft's speed during the first 30 seconds or so of launch will not be fast enough to impart an adequate frequency difference to the return light and thus another method of wavefront detection is required. This paper discusses the viability of using a nanocraft-releasing mothercraft satellite with an on-board laser guide star as a beacon for wavefront sensing. In order to assess if this beacon satellite would offer a viable wavefront sensing method, we explored orbital solutions that would keep the mothercraft within the isoplanatic angle of the nanocraft during the critical 30 second window. A parameterized astrodynamics model was created to track the angular separation of both objects in the sky as a function of time, arbitrary orbital parameters, and the laser's radiation force vector. We found that for a sufficiently large orbital semi-major axis there exists an optimized orbital eccentricity which reduces the angular separation of the mothercraft and nanocraft in the sky to less than the isoplanatic angle during the first 30 seconds of launch. We also found that focal anisoplanatism was negligible during that period at the distance of the mothercraft's apogee.

Keywords: Astrodynamics, Adaptive Optics, Isoplanatic Angle, Focus Anisoplanatism, Wavefront Sensing

1. INTRODUCTION

The Breakthrough Starshot Initiative plans to use the pressure of laser light on the solar-sail of a nanocraft to accelerate it to 20% the speed of light (c) over the course of 10 minutes[1]. In order for this laser projector to efficiently transfer momentum from its photons to the spacecraft, the beam needs to be well aimed and focused onto the nanocraft. However, because the beam has to travel through Earth's atmosphere this focus will spread and its aim will deflect, causing most of the light to miss its target. To mitigate this problem, a method for wavefront sensing and prelaunch AO correction is required. We anticipate that the primary method for wavefront sensing will be to use the reflected photons from the nanocraft as a beacon source. To disambiguate these reflected photons from those scattered off the atmosphere, they will have to be Doppler shifted by the nanocraft's motion, so an acceleration period of 30 seconds must pass to reach a speed for a sufficient Doppler shifted return. Therefore, a separate wavefront sensing method must be used during this 30 second period.

Our proposed solution for this period is to use a satellite-based laser guide star. This satellite would be referred to as the "mothercraft" because it would house the nanocraft for release into orbit at the moment of launch. As the launch projector activates and begins accelerating the nanocraft away, the mothercraft would be emitting a beacon of a different frequency to be used for wavefront sensing (Figure 1). For this method to work, it is necessary that the configuration of the mothercraft and nanocraft satisfy two primary specifications: 1) relative to the nanocraft, the mothercraft's angular separation in the sky must remain within a specified limit known as the isoplanatic angle, and 2) the height of the beacon should be high enough that it adequately samples the same volume of atmosphere the projection beam will travel through. These two specifications are referred to as angular and focal anisoplanatism, respectively (Figure 2).

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To determine if this proposed method meets these criteria, a parameterizable astrodynamics model was created to track the 3D positions of the nanocraft, mothercraft, and launch projector as functions of time during this 30 second launch period. This modelled data allowed us to predict the error contribution expected in the system, and to optimize the mothercraft's orbital parameters to minimize these errors.

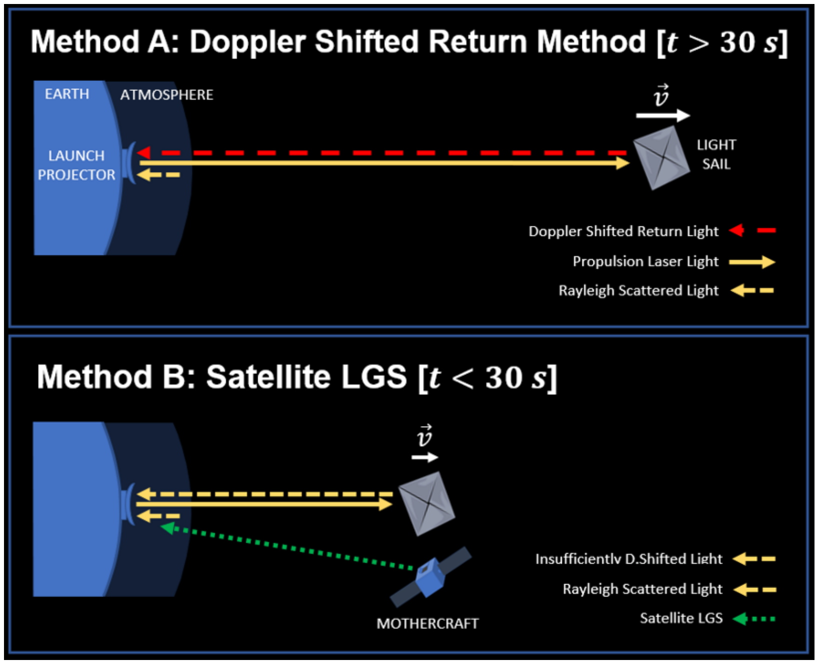

Figure 1. Concepts for the two wavefront sensing methods.

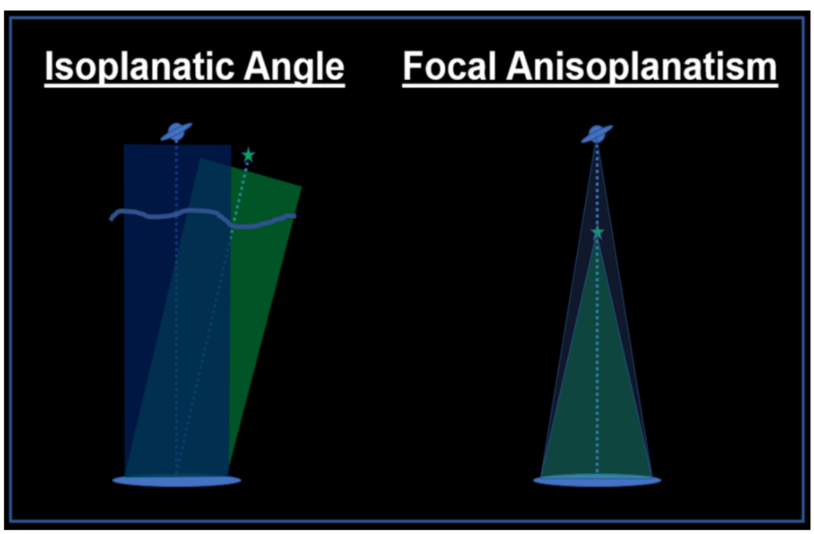

Figure 2. Performance Criteria: Above are the two geometries between a beacon source and a target of interest that determine the expected performance quality of an AO system.

2. ORBITAL MODELING

2.1 Model Architecture

In order to determine if the separation of the mothercraft and the nanocraft is larger than the isoplanatic angle after 30 seconds, knowledge of the mothercraft's orbital motion (Figure 3) and nanocraft's trajectory are required. The orbit of the mothercraft will determine the component of its velocity transverse to the line of sight following the release of the nanocraft and thus its angular separation versus time. There are two fundamental orbital characteristics desirable for minimal separation: long orbital period, and high eccentricity. Assuming release at apogee, an orbit with a long orbital period and a high eccentricity will yield a smallest tangential velocity. In order to determine which orbits yield an object angular separation less than the isoplanatic angle θ_0, a fully parameterizable orbital simulator was created that calculates the position and velocity over time of a satellite given any arbitrarily assigned orbital parameters. There were two methods used for modeling the orbits/positions of the mothercraft and nanocraft: A) Keplerian Modeling, and B) Relativistic Numerical Modelling. The first is used to model the position of the *mothercraft*, and the second is to model the motion of the nanocraft. The second model considers the time of flight of the propulsion light and the relativistic effects the speed has in force coupling.

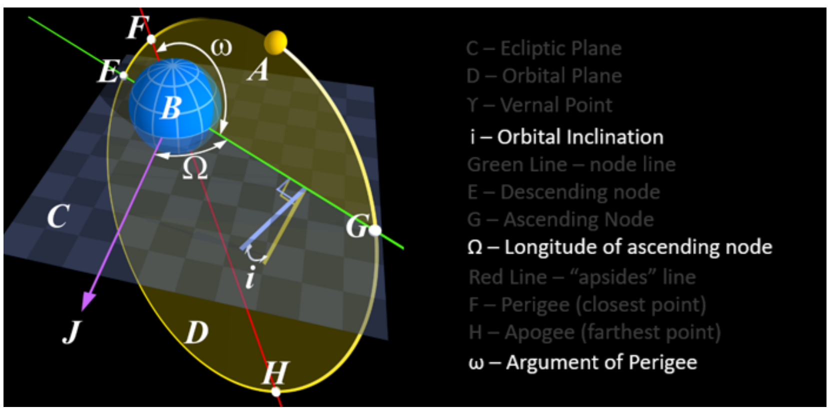

Figure 3. Key Orbital Parameters

2.1.1 Keplerian Modeling

For the position of the mothercraft in the sky we used classical Keplerian modeling (Figure 4). This modeling method was chosen because the mothercraft's motion is non-relativistic and because a Keplerian orbital model only takes four mathematical steps to determine a satellite's position as a function of time[2]. This means it is not computationally intense. Also, its input parameters are basic orbital parameters that can be defined by the user. This makes it very easy to optimize the mothercraft's orbit to minimize anisoplanatic errors. The Keplerian model assumes two things: 1) that the satellite mass is much smaller than the body it orbits, and 2) that no forces other than gravity are acting on the satellite.

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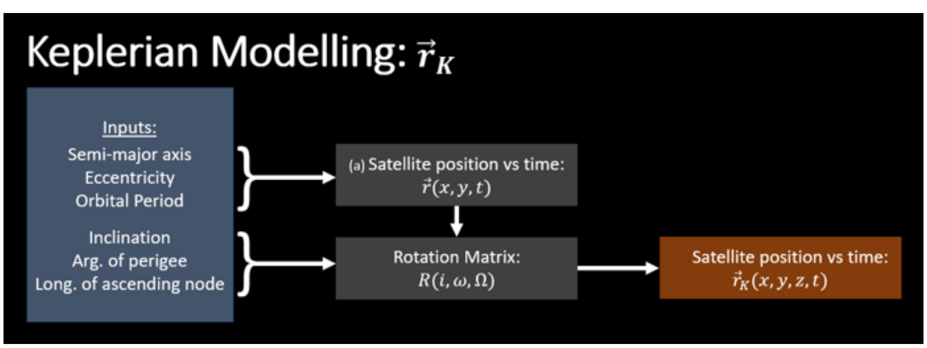

Figure 4. The Keplerian Orbital Model Diagram

2.1.2 Relativistic Numerical Modeling

For the nanocraft, another method for modeling its position in orbit was needed (Figure 5). This is because the nanocraft changes its orbit dynamically due to the applied force of the launch laser which violates the second assumption of Kepler's equations. Additionally, the nanocraft reaches speeds that are appreciable fractions of the speed of light. These speeds introduce relativistic effects on the nanocraft's acceleration that must be accounted for. One of the most important of these effects is the change in force caused by the Doppler shift of the incoming light (from the perspective of the nanocraft).This effect, as well as the finite speed of light, must be account for in order to model the change in the nanocraft's position dynamically. Using sufficiently small time intervals, the nanocraft's rest-frame acceleration is calculated numerically. During each time interval, the nanocraft's velocity and position can be integrated from that interval's acceleration.

$$\vec{a}(t) = \vec{F}_{total}(t) * m_{nanocraft} \tag{1}$$

The total force acting on the nanocraft is initially comprised of two forces: gravity, and the force of the laser (radiation pressure). The gravitational force is determined by the nanocraft's position relative to earth's center. Because the nanocraft escapes the sphere of influence of Earth's gravity before it reaches relativistic velocities, gravity's influence on the nanocraft is calculated as a classical Newtonian force. The rapid departure of the nanocraft also means that the force of gravity rapidly drops to be a negligible contribution to the nanocraft's acceleration. The force of the laser is determined by its position relative to the launch projector and the relativistic velocity of the nanocraft (β).

$$\beta = \frac{v}{c} \tag{2}$$

$$P_b = \eta_a \eta_b P_0 \tag{3}$$

$$P_s{}' = \frac{1-\beta}{1+\beta} P_b \tag{4}$$

$$F_{Laser} = \frac{P_s'}{c} \tag{5}$$

$$\vec{a}(t) = [\vec{F}_{gravity}(t) + \vec{F}_{Laser}(t, \beta)] * m_{nanocraft} \tag{6}$$

P_0 is the power leaving the launch aperture, and P_b is the power that reaches the spacecraft. η_a is the atmospheric transmission along the beam path, and η_b is the transfer efficiency of the beam as defined by Kulkarni et al (2018). η_b accounts for the diffraction of a Goubau beam of wavelength λ_0 leaving an aperture of size D_b a distance $|r|$ to a sail of size D_s[3]. The overall launch geometry is illustrated in Figure 6.

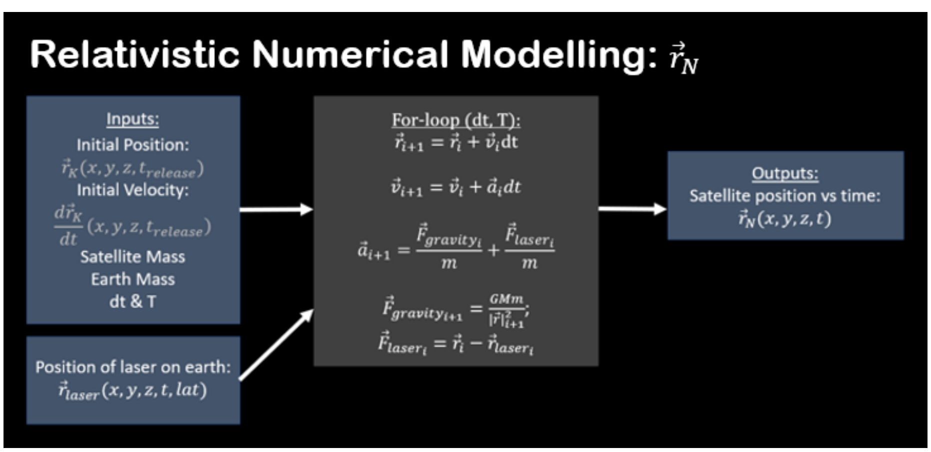

Figure 5. The Relativistic Numerical Orbital Model Diagram

*The position of the launch projector on earth's surface ($\vec{r}_{Laz,i}$) was also parameterized. Given its latitude and time of day, its position under the nanocraft, on earth, as the earth rotated, could be determined at each time step

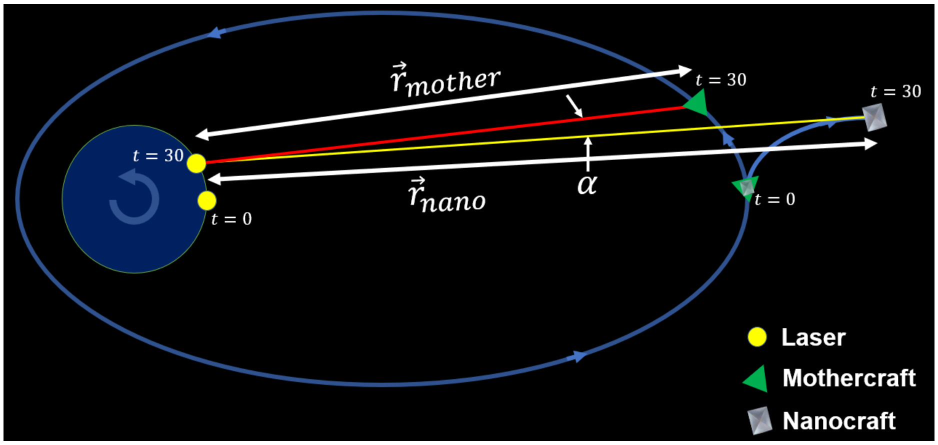

Figure 6. Geometrical depiction of the model outputs used in performance characterization.

2.1.3 Photon Time of Flight and Aim Leading

Because light has a finite speed it will take time for the propulsion beam to reach its target, the nanocraft. The Relativistic Numerical Orbital Model accounts for this by *leading* its target. Given the initial positions of the launch projector ($\vec{r}_{Laz,0}$) and the nanocraft ($\vec{r}_{nano,0}$), the initial velocity of the nanocraft ($\vec{v}_{nano,0}$), and the speed of light (c), the *leading direction* of the launch projector ($\hat{L}_{Laz}$) can be determined to assure the light impacts the nanocraft at an impact location ($\vec{P}_{impact}$). The time of flight between the laser projector and the location of impact is given as

$$t_{TOF} = \frac{\sqrt{(\vec{P}_{impact,x} - \vec{r}_{Laz,x})^2 + (\vec{P}_{impact,y} - \vec{r}_{Laz,y})^2 + (\vec{P}_{impact,z} - \vec{r}_{Laz,z})^2}}{c} = \frac{distance}{speed} \tag{7}$$

In order for the impact to occur, the nanocraft must have traveled along its path for this same amount of time to reach the point of impact when the beam does

$$.\vec{r}'_{nano} = \vec{P}_{impact} = \vec{r}_{nano,0} + \vec{v}_{nano,0} t_{TOF}$$

Expressed component by component:

$$\vec{P}_{impact,x} = \vec{r}_{nano,0,x} + \vec{v}_{nano,0,x} t_{TOF} \tag{9}$$

$$\vec{P}_{impact,y} = \vec{r}_{nano,0,y} + \vec{v}_{nano,0,y} t_{TOF} \tag{10}$$

$$\vec{P}_{impact,z} = \vec{r}_{nano,0,z} + \vec{v}_{nano,0,z} t_{TOF} \tag{11}$$

Plugging in equations 9-11 into equation 7 and rearranging gives:

$$t^2 c^2 = (\vec{r}_{nano,0,x} + \vec{v}_{nano,0,x} t_{TOF} - \vec{r}_{Laz,x})^2 + (\vec{r}_{nano,0,y} + \vec{v}_{nano,0,y} t_{TOF} - \vec{r}_{Laz,y})^2$$
$$+ (\vec{r}_{nano,0,z} + \vec{v}_{nano,0,z} t_{TOF} - \vec{r}_{Laz,z})^2$$

Expanding and rearranging the above expression yields:

$$0 = \left[\vec{v}_{nano,0,x}^2 + \vec{v}_{nano,0,y}^2 + \vec{v}_{nano,0,z}^2 - c^2\right] * t_{TOF}{}^2$$
$$+[2 * \left(\vec{v}_{nano,0,x}\left(\vec{r}_{nano,0,x} - \vec{r}_{Laz,x}\right) + \vec{v}_{nano,0,y}\left(\vec{r}_{nano,0,y} - \vec{r}_{Laz,y}\right) + \vec{v}_{nano,0,z}\left(\vec{r}_{nano,0,z} - \vec{r}_{Laz,z}\right)\right] * t_{TOF}$$
$$+ \left[\left(\vec{r}_{nano,0,x} - \vec{r}_{Laz,x}\right)^2 + \left(\vec{r}_{nano,0,y} - \vec{r}_{Laz,y}\right)^2 + \left(\vec{r}_{nano,0,z} - \vec{r}_{Laz,z}\right)^2\right]$$

The above is of the quadratic form: $Ax^2 + Bx + C = 0$, with

$$A = \left|v_{nano,0}\right|^2 - c^2 \tag{12}$$

$$B = 2 * \left[v_{nano,0} \cdot \left(\vec{r}_{nano,0,} - \vec{r}_{Laz,0}\right)\right] \tag{13}$$

$$C = \left|\left(\vec{r}_{nano,0,} - \vec{r}_{Laz,0}\right)\right|^2 \tag{14}$$

The above three equations can be plugged into the quadratic equation to find the time of flight necessary for coincident impact. If the discriminant of the quadratic equation is negative, then there is no real solution. Fortunately, this primarily occurs when the velocity of the target (the nanocraft) is faster than the velocity of the projectile (light), which is impossible for our system. For our system, the solution will include both positive and negative solutions for time of flights. The physical solution is taken as the positive one. This solution can then be plugged back into equations 9-11 to find $\vec{P}_{impact}$. The leading direction, $\hat{L}_{Laz}$, can be found with the following equation:

$$\hat{L}_{Laz,0} = (\vec{P}_{impact} - \vec{r}_{Laz,0}) / \left|\vec{P}_{impact} - \vec{r}_{Laz,0}\right| \tag{15}$$

This leading direction is then used to determine the direction of the force the laser beam imparts onto the nanocraft.

However, Equation 15 is only useful for the very first point in the numerical model. This is because one can specify $\vec{r}_{Laz,0}$, $\vec{r}_{nano,0}$, and $\vec{v}_{nano,0}$. In order to determine $\hat{L}_{Laz,i}$, where i is some interval within the numerical analysis, one must know how the force of the beam from $\hat{L}_{Laz,i-1}$ affected $\vec{r}_{nano,i-1}$, and $\vec{v}_{nano,i-1}$; however, because the length of the numerical interval (sub = 1ms) is much shorter than the expected time of flights ($t_{TOF,i} > 1s$) the photons from $\hat{L}_{Laz,i}$ will have already been traveling for $(t_{TOF,i} - sub)$ by the time the nanocraft reaches $\vec{P}_{impact,i-1}$. The solution is to take the position of the nanocraft at a given interval ($\vec{r}_{nano,i}$) and use its velocity at that same time interval ($\vec{v}_{nano,i}$) to determine where it would have been $(t_{TOF,i} - sub)$ seconds ago. This dummy position, $\vec{r}_{nano,i}{}'$, and the unaltered $\vec{v}_{nano,i}$ can then be used as the 'initial position and velocity' for each interval calculation of $\hat{L}_{Laz,i}$ the same way that $\vec{r}_{nano,0}$ and $\vec{v}_{nano,0}$ were used to calculate $\hat{L}_{Laz,0}$ with equations 7-15.

2.1.4 System Performance Characterization

With the 3-dimensional positions of the mothercraft, nanocraft, and laser projector established, the anisoplanatic contributions to the wavefront estimation error can be calculated to characterize the expected performance of the beam compensation system. To determine the wavefront variance attributed to *angular* anisoplanatism, the following equation is used where the units are rad^2.[3]

$$\sigma_{iso}^2 = \left(\frac{\alpha}{\theta_0}\right)^{5/3} \tag{16}$$

where α is the angular separation in the sky between the mothercraft and nanocraft, and θ_0 is the isoplanatic angle at the beacon wavelength of 1064 nm. For our system and wavelength, θ_0 is taken to be approximately 20 μrad. α can be determined at each time step by using the unit vectors that point from the launch projector to both the mothercraft and nanocraft ($\widehat{r_m}$ & $\widehat{r_n}$ respectively). Using the definition of a dot product the angular separation is found as

$$\alpha(t) = \text{acos}\left(\frac{\widehat{r_m}(t)}{\widehat{r_n}(t)}\right) \tag{17}$$

To determine the wavefront variance attributed to *focal* anisoplanatism, the following equation is used[3]

$$\sigma_{FA}^2 = \left(\frac{D}{d_0}\right)^{\frac{5}{3}}\left(1 - \frac{|\vec{r}_{mother}|}{|\vec{r}_{nano}|}\right)^{\frac{5}{3}} \tag{18}$$

where *D* is the aperture diameter of the launch projector (2 km), d_0 is a characteristic length scale determined by the atmospheric C_n^2 profile and the beacon range, and $|\vec{r}_{mother}|$ & $|\vec{r}_{nano}|$ are the distances to the mothercraft and nanocraft [in km]. Note that Eq. 4 is just the classical form derived by Fried[4] but with a correction factor. This factor accounts for the fact that the object of interest, the nanocraft, is not located infinitely far away (which the classical form assumes). The additional term respects two important boundary conditions: 1) $\sigma_{FA}^2 = 0$ when the nanocraft and mothercraft are coincident in space, thus no focal anisoplanatism, and 2) $\sigma_{FA}^2 = \left(\frac{D}{d_0}\right)^{5/3}$ when the nanocraft approaches infinity (in which we find the classic formula). The power of 5/3 is used to keep the appropriate scaling in relation to the atmospheric turbulence. Lastly, d_0 is assumed to also vary as the mothercraft's altitude changes, and is determined assuming a conservative Hufnagel-Valley 5/7 model[4]

$$d_0 = 0.018|\vec{r}_{mother}| + 0.39\ [m] \tag{19}$$

with the equation input, $|\vec{r}_{mother}|$, in units of km.

2.1.5 Parameter Optimization

With the completed model for determining the viability of a given orbit for wavefront sensing, an optimizer was created that looped through a series of potential orbits to find those which yielded a value $\alpha < \theta_0$ after 30 s. Because the nanocraft's aim point in the direction of Proxima Cen is at a fixed point on the celestial sphere, the *orbit's inclination, longitude of ascending node,* and *argument of perigee* ($i, \Omega, \& \ \omega$) are pre-determined. Therefore, the variables of the orbit that can be optimized for the AO system are: *Semi-major axis* [SMA] (which defines orbital period), *orbital eccentricity* [ϵ], *time of release* (how many seconds before/after apogee is the nanocraft released), *time of launch* (how many seconds before/after the point directly below apogee is the laser in its rotation on earth), and *latitude* (of the launch projector on earth).

There are some further bounds on these variables. Because semi-major axis defines orbital period, we must only use those that yield orbital periods that are an integer multiple of 12 sidereal hours. This assures that at least one nanocraft clone can be launched every day.* Also, the eccentricity should not be so high that the orbit's perigee intersects Earth's atmosphere which would lead to a rapid loss of the mothercraft to atmospheric drag. With these two constraints, we ran the model for every combination of the variables given in Table 1. Note that the range of latitude is selected to span the Atacama plateau, which, while not extending south to the ideal latitude, offers already developed high-altitude sites suitable for construction of the projector.

Parameter	Range of Values	Step Size
Orbital Period	[12hrs, 96hrs]	12hrs
Eccentricity	[0, 0.99]	0.001
Latitude	[-18, -30]	0.1
Time of Launch	[-60, 60]	1
Time of Release	[-60, 60]	1

Table 1. Optimization Parameters

*By sending many clones of the nanocraft towards Proxima Cen b, the Starshot Program increases the statistical chances of mission success the same way insects ensure future reproduction by having many offspring.

2.2 Optimization and Model Results

Following optimization, we found that for every SMA, latitude, and eccentricity the optimal t_{launch} and $t_{release}$ were always minimized. Latitude was also optimally as close to the orbital inclination as possible. Therefore, for each input SMA, there was a range of eccentricities that kept $\alpha < \theta_0$. We also found that as both the orbital period and eccentricity increase the angular separation between the mothercraft and nanocraft decreased. This follows with the expectation for the optimal orbit.

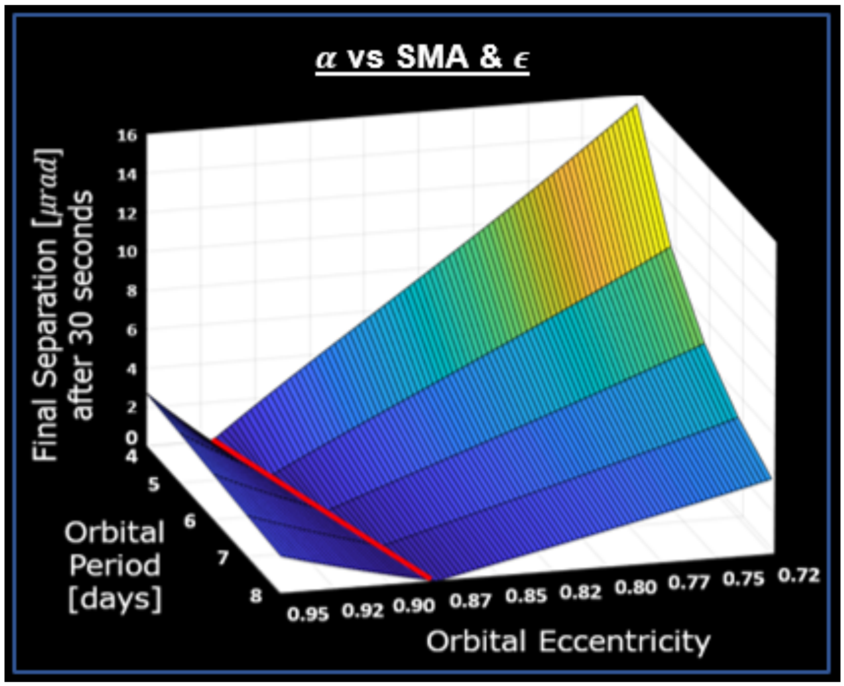

Figure 7. Optimization results: Finding how final separation is affected by SMA and ϵ.

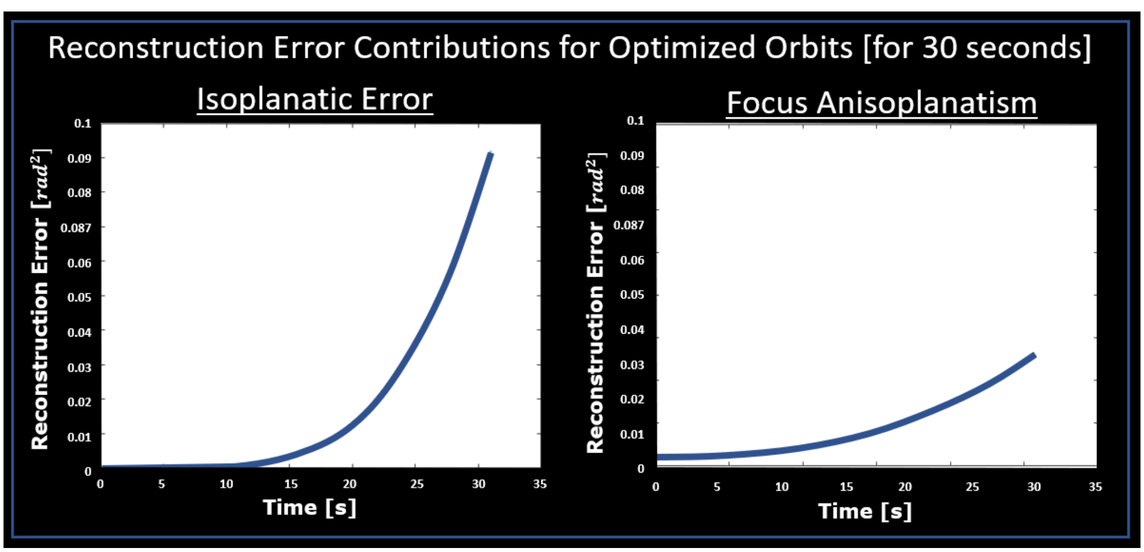

Figure 8. Expected system performance given optimized orbital parameters.

In Figure 7 above, the final angular separation of the mothercraft and nanocraft in the sky from the perspective of the launch projector is seen as a 3D plot with SMA and ϵ as the independent variables. The portions of this surface that remain below $\theta_0 = 20\ \mu$rad are the optimized parameters that satisfy the performance criteria of the AO correction. Choosing the smallest SMA so as to maximize the frequency of nanocraft launch, the final optimized orbital parameters for the Breakthrough Starshot launch assuming the use of a mothercraft mounted laser guide beacon for wavefront correction are

SMA | Orbital Period	106 Mm | 96 hours (4 days)
Eccentricity [ϵ]	0.9172
Orbital Inclination [i]	-62 degrees
Latitude of Projector	-30 degrees

Table 2. Final Optimal Orbital Parameters for Breakthrough Starshot's Mothercraft Laser Guide Beacon

Taking these optimal orbital parameters and running them through the system performance characterization portion of our code, we found the residual wavefront error expected from both angular and focal anisoplanatism. In Figure 8, we see that given these optimal orbital parameters the total error expected following adaptive correction is less than 0.15 rad^2. This result stems from two characteristics of the orbit. The first is that because the orbit's apogee and eccentricity are so large, the tangential velocity of the mothercraft is small enough that it does not separate much over a 30 second period. The second is that the large altitude of apogee means that the volume of atmosphere traversed by returning beacon light does not differ substantially from the volume through which the outgoing laser light passes. The wavefront error following correction will not degrade the focused beam's final Strehl ratio by more than 85%. We believe therefore that this method of using the mothercraft as a beacon source for wavefront sensing over the initial launch period *is viable*.

3. SUMMARY AND FUTURE IMPROVEMENTS

Breakthrough Starshot is program which aims to send mankind's first space probes to another star system. To achieve this, nano spacecraft will be launched using a high-powered ground-based laser projection system which will use radiation pressure to accelerate each probe to 20% the speed of light. In order for this beam to properly focus precisely onto the spacecraft through the turbulence of the atmosphere a pre-correction from an adaptive optics system is required. In order to determine what pre-correction must be applied to the beam, a measurement of the atmosphere's turbulence must be made. This wavefront sensing measurement can in principle be performed with several different techniques. The primary method we propose relies on the Doppler shifted return light from the nanocraft as a beacon for wavefront sensing. However, during the first 30 seconds of acceleration, the nanocraft will not be traveling fast enough for the light to be sufficiently Doppler shifted and segregable from the Rayleigh scattered light returning from the atmosphere. An alternative method for wavefront sensing during this period is to use the light from a laser beacon aboard the nanocraft's mothercraft. For this method of wavefront sensing to be viable, the mothercraft's angular separation from the nanocraft must remain within the system's isoplanatic angle and be similar enough in altitude to that of the nanocraft that each beam sees the equivalent volume of atmosphere.

To test this method's viability, a parameterizable astrodynamics model was created for the 3D positions of the mothercraft and nanocraft relative to the launch projector. These parameters included the position of the launch projector on earth, the orbital parameters of the mother craft, and the characteristics of the launch laser's activation in relative time. The outputs of this model could be used to then optimize its own input parameters to determine what values maximized the system's expected performance.

Following optimization and modelling, it was found that the optimal orbital parameters from the mothercraft that adequately reduced its angular separation from the nanocraft during the 30 second launch period were: SMA = 106 Mm, ϵ = 0.9160, Orbital Inclination [i] = -62 degrees (given that the launch projector was located at a latitude of -30 degrees in the Atacama desert). With these optimal orbital parameters, it was found that the expected residual wavefront error following adaptive correction would be 0.15 rad^2. This residual error will be lower than other sources, in particular residual high-order aberration left uncorrected by the AO system which is likely to be at least 100 nm rms, or 0.35 rad^2. Use of the mothercraft as a platform for a laser beacon therefore appears viable during the initial launch window.

Future work will include the model's analysis of the system's performance given a longer time of use. The idea of this analysis would be to see if this method of wavefront sensing could be viable for use during *the entire* 10 minute launch period. If confirmed viable, this would reduce the Starshot System's overall complexity by only needing one system for wavefront sensing instead of two. Another future analysis would include how imperfect adaptive correction could lead to variations in the applied force of the sail.

ACKOWLEDGMENTS

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