Atmospheric tomography with pyramid wavefront sensors for spiders

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ABSTRACT

The new generation of Extremely Large Telescopes (ELTs) have thick holding structures for the secondary mirror - also known as spider legs. The incoming light is obstructed by these spider legs and therefore the sensor measurements are split into segments. This disconnection of the data results into piston segments in the wavefront reconstruction which effect the quality severely. The pyramid sensor has the advantage to be more sensitive to local piston modes. Therefore it is the wavefront sensor of choice for many currently built instruments. In their development the focus is on single conjugated adaptive optics (SCAO). Future systems of the adaptive optics will also contain tomographic modes like laser tomography (LTAO) or multi conjugated adaptive optics (MCAO). We use a wavelet based iterative method (FEWHA) for atmospheric tomography, which is adapted to a linearized pyramid model. We present end-to-end simulation results for the SCAO case in a METIS like setting. This case is extended to achieve a tomographic system, intermediate results are shown.

Keywords: pyramid wavefront sensor, linearized roof sensor, atmospheric tomography, adaptive optics, wavelets

1. INTRODUCTION

For the next generation of extremely large telescopes the control of the adaptive optics (AO) systems is challenging, due to a large number of actuators to be controlled in real-time as well as real-life phenomena like the island effect. It is well known that the pyramid wavefront sensor has multiple advantages over the Hartmann-Shack sensor. In particular, it is more sensitive to local piston measurements, which makes it more effective to overcome the spider problem, also known as the island effect. For the Hartmann-Shack wavefront sensor there exists an efficient method for atmospheric tomography known as the Finite Element-Wavelet Hybrid Algorithm (FEWHA[1]). We develop several strategies to adapt the Finite Element-Wavelet Hybrid Algorithm to be able to handle pyramid wavefront sensor measurements. One method is to transform the pyramid data to Hartmann-Shack measurements. The method works good, but it is suffering the Island effect, i.e., a further investigation is necessary as done in this papers [2,3].

Another option is to include a forward model of the pyramid sensor, instead of the Hartmann-Shack sensor. As a first step we used a linearization of the roof sensor, where the roof sensor is already an approximation of the pyramid sensor. In this case we call the method *FEWHA 4 pyramid*. We will demonstrate the quality of our algorithm in the context of a single conjugated adaptive optics (SCAO) system. We also show an intermediate result for a tomographic AO system, handling the pyramid wavefront sensor.

In Section 2 we give a brief overview of the FEWHA method. Section 3 describes the modification of the FEWHA algorithm for the pyramid sensor and we show arising difficulties by the non-linearity nature of the sensor, since the FEWHA method is linear. The numerical experiments are documented in Section 4 and in Section 5 we state our conclusions.

2. FINITE ELEMENT-WAVELET HYBRID ALGORITHM (FEWHA)

The FEWHA is a conjugate gradient based approach which computes the MAP estimate of wavefront-to-wavefront-sensor measurement equations in the Bayesian framework. It combines a finite element and a wavelet basis to solve the atmospheric tomography problem, including atmospheric statistics, in an efficient way. FEWHA was originally designed for Hartmann-Shack wavefront sensors and in that setting the runtime complexity is of $\mathcal{O}(n)$, where n is the number of actuators to be controlled.

Consider the problem of atmospheric reconstruction in the random variables setting,

$$\mathbf{S} = \mathbf{\Gamma}\mathbf{P}\mathbf{\Phi} + \mathcal{E},$$

where the random variables $\mathbf{S}$, $\mathbf{\Phi}$, $\mathcal{E}$ are *measurements*, *turbulence layers* and *noise*. Assume $\mathbf{\Phi}$ and $\mathcal{E}$ are Gaussian with zero expectation and covariance operators $C_{\mathbf{\Phi}}$ and $C_{\mathcal{E}}$. The Hartmann-Shack wavefront operator is denoted by $\mathbf{\Gamma}$ and $\mathbf{P}$ is the projection of the atmospheric layers to the ground layer.

By a dual-domain discretization of the Bayesian MAP estimate, assuming the Kolmogorov turbulence model and using the Bernstein-Jackson Inequalities, the wavelet coefficients $\mathbf{c}$ are given as a solution to

$$(\mathbf{W}\mathbf{P}^{*}\mathbf{\Gamma}^{*}\mathbf{C}_{\mathcal{E}}^{-1}\mathbf{\Gamma}\mathbf{P}\mathbf{W}^{-1} + \alpha\mathbf{D})\mathbf{c} = \mathbf{W}\mathbf{P}^{*}\mathbf{\Gamma}^{*}\mathbf{C}_{\mathcal{E}}^{-1}\mathbf{s}. \tag{1}$$

The benefit of the wavelet domain is the approximately representation of the atmospheric statistics C_ϕ in a diagonal matrix, while the wavelet transform itself is of linear complexity.

3. FEWHA FOR PYRAMID

The original design of the FEWHA was for the Hartmann-Shack wavefront sensor. Therefore, not only the sensor model, but also the hole method is linear. In contrast, the pyramid wavefront sensor maps the incoming wavefront to the measurements in a non linear way. To get a first impression of the achievable quality, we use a linearization of the roof sensor, which is as an approximation of the pyramid sensor. This linearization of the roof sensor in x-direction is given by

$$\left(R_x^{\{c\},lin}\phi\right)(x,y) := \mathcal{X}_{\Omega_y\times\Omega_x}(x,y)\frac{1}{\pi}\,p.v.\!\int_{\Omega_y}\frac{\left(\phi(x',y)-\phi(x,y)\right)J_0(\alpha_\lambda(x'-x))}{x'-x}dx', \tag{2}$$

similarly in y-direction.

Replacing the Hartmann-Shack operator Γ in the Finite Element-Wavelet Hybrid Algorithm, equation (1), by the linearized roof sensor $\mathbf{R} = \left(R_x,\, R_y\right)^T$ we obtain

$$(\mathbf{W}\mathbf{P}^{*}\mathbf{R}^{*}\mathbf{C}_{\mathcal{E}}^{-1}\mathbf{R}\mathbf{P}\mathbf{W}^{-1} + \alpha\mathbf{D})\mathbf{c} = \mathbf{W}\mathbf{P}^{*}\mathbf{R}^{*}\mathbf{C}_{\mathcal{E}}^{-1}\mathbf{s}, \tag{3}$$

where $\mathbf{R}^*$ is the adjoint operator. The numerical complexity of the FEWHA for Hartmann-Shack operator is of $\mathcal{O}(n)$, where n is the number of active subapertures. Since the application of the linearized roof sensor is of $O(n\sqrt{n})$[2], the numerical effort for *FEWHA 4 pyramid* also increases to $O(n\sqrt{n})$.

The FEWHA contains only linear operators, i.e., the hole method is linear. If the non-linear nature of the pyramid sensor is of importance, then the following operations need a further investigation:

- Pseudo Open Loop Control (POLC)
  FEWHA uses atmospheric statistics and therefore it needs open loop data. But since the usual operating mode is in closed loop, we need to achieve open loop data. A solution is the Pseudo Open Loop Control (POLC), which is applicable only for linear operators as the Hartmann-Shack operator Γ. For simplification we consider only the SCAO case. The pseudo open loop wavefront φ^{POLC} is split into the previously reconstructed wavefront φ^{OLD} and its update φ^{UPDATE}. Then the rough idea of computing pseudo open loop data $s^{\text{POLC}}$is the following,

$$s^{\text{POLC}} = \mathbf{\Gamma}\phi^{\text{POLC}} = \mathbf{\Gamma}(\phi^{\text{OLD}} + \phi^{\text{UPDATE}}) = \mathbf{\Gamma}\phi^{\text{OLD}} + \mathbf{\Gamma}\phi^{\text{UPDATE}} = \mathbf{\Gamma}\phi^{\text{OLD}} + s^{\text{CL}}$$

So the computation of s^{POLC} demands the computation of generic measurements $\mathbf{\Gamma}\varphi^{\mathrm{OLD}}$. In case of non-linear operators the upper equation is not valid.

- Conjugate gradient method (CG)
  The CG method is an algorithm for solving a system of linear equations. By the use of the non-linear pyramid sensor, the numerical discretized equations (3) to be solved, are also not linear anymore.

4. NUMERICAL RESULTS

The FEWHA requires a bilinear representation of the wavefront. Therefore, a simple discretization of (2) is done in the bilinear domain. First we show numerical results for SCAO in a METIS like setting and we compare it to other reconstructors. The pupil mask used for M1 applied to 6 spider legs is illustrated in Figure 1. Secondly, we simulate a tomographic case by copying the same guide star to an asterism. We present intermediate results for this natural guide star tomography. In this setting we compare our results to the Hartmann-Shack case. All simulations are performed on the European Southern Observatory (ESO) simulation environment called OCTOPUS.

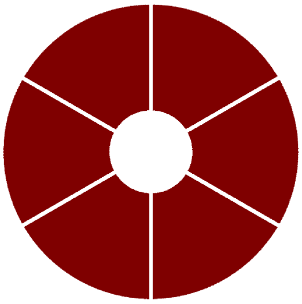

Figure 1. Spider mask with 6 legs.

4.1 Single Conjugated Adaptive Optics

The basic configuration for the SCAO case is given in Table 1. The application of *FEWHA 4 pyramid* is straight forward, no play around with parameters like the illumination factor of the subapertures is needed. The parameters for the reconstruction method are shown in Table 2. The Strehl evaluation shows a satisfying handling of the island effect, see Figure 2. The result is compared in Table 3, it shows the promising quality of the reconstructor. The other methods, which are capable to handle the spider effect are described here [2,3].

Table 1. OCTOPUS configuration parameters of SCAO system

telescope diameter	37m
subapertures	74
spider legs	6
with thickness	0.5m
ESO standard median atmosphere	$r_0 = 0.157$
number of layers	35
modulation of pyramid wavefront sensor	$4\frac{\lambda}{D}$
sampling frequency	500Hz
photons/subaperture/frame	600
number of loop of iterations	1000
M4 geometry	5190 actuators
evaluation and sensing in K-band	$\lambda = 2200$nm

Table 2. *FEWHA 4 pyramid* parameter setting

regularization parameter	1
number of CG iterations	20
gain	0.3

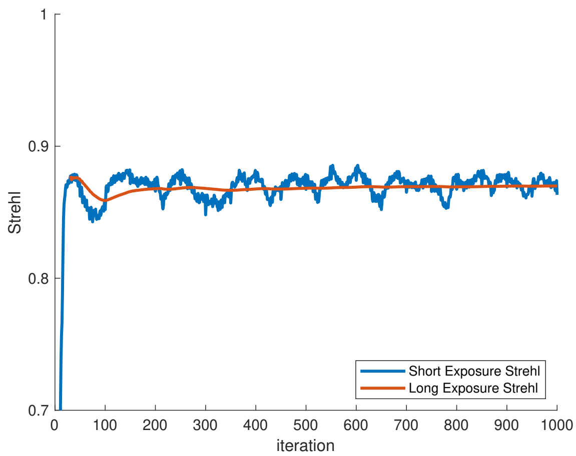

Figure 2. Long and short exposure Strehl ratio for SCAO.

Table 3. Comparison of different methods.

Reconstruction methods	Strehl
No spiders	0.8851
DSPR I	0.8775
DSPR II	0.8654
Zonal MMSE	0.88
FEWHA	0.8698

4.2 Natural Guidestar Tomography

In this Section we show first results for atmospheric tomography with pyramid sensors. Note, that this work is ongoing and all the results are intermediate without extensive numerical optimization. Since FEWHA has a tomography mode integrated, also more specific AO modes are possible. We avoid the use of a laser guide star system and concentrate on natural guide star asterism as shown in Figure 3. The separation of the guide stars is $0.8'$. The wavefront sensor setting of each guide star is the same as for the SCAO case, see Table 1. Due to numerical complexity of simulated guide star system, the number of loop iterations is reduced to 500. The comparison to the Hartmann-Shack wavefront sensor is shown in Figure 4 and in Table 4. In both cases, with and without spiders, we observe a loss of quality towards the Hartmann-Shack operator. It has not been determined yet if this tomographic error is due to the effects of linearization of the pyramid sensor or if numerical optimization would improve the Strehl ratio.

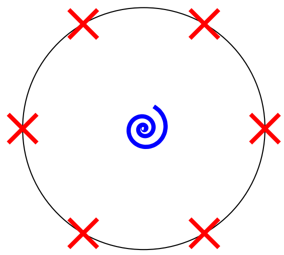

Figure 3. Natural guide stars in an asterism with 0.8 arcminutes separation and the centered object of interest.

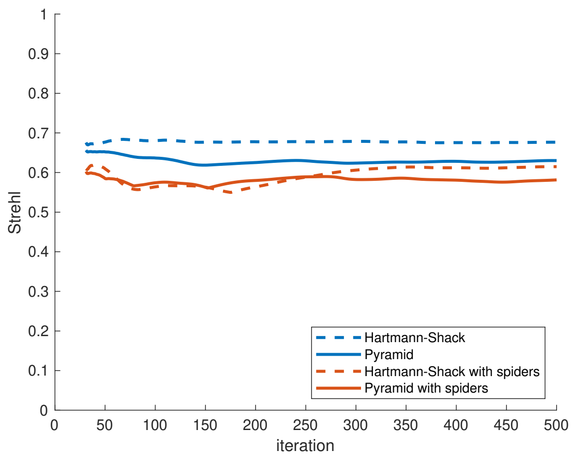

Figure 4. Strehl evaluation of linearized roof and Hartmann-Shack wavefront sensor in tomographic mode, with and without spider legs.

Table 4. Compare Long Exposure Strehl of linearized roof sensor to the Hartmann-Shack sensor.

	without spiders	with spiders
Hartmann-Shack	0.6765	0.6148
Pyramid	0.6302	0.5814

5. CONCLUSIONS

There is a need for developing a high performance reconstruction method which can handle the standard configurations of an AO system like MCAO, LTAO, SCAO, etc. equipped with pyramid sensors. We have extended the FEWHA method by adding a linearized roof sensor to the *FEWHA 4 pyramid*. First results are promising and motivate for further research. Its numerical complexity increases then to $\mathcal{O}(n\sqrt{n})$, where n is the number of active actuators to be controlled. In the SCAO case the method already delivers good quality. For the tomography, further research is necessary.

REFERENCES

[1] M. Yudytskiy, T. Helin, R. Ramlau: *Finite element-wavelet hybrid algorithm for atmospheric tomography.* J. Opt. Soc. Am. A, 31(3):550-560, Mar 2014.

[2] V. Hutterer et al. *Advanced wavefront reconstruction methods for segmented ELT pupils using pyramid sensors.* J. of Astronomical Telescopes, Instruments, and Systems, 4(4), 049005 (2018).

[3] A. Obereder et al. *Dealing with Spiders on ELTs - using a Pyramid WFS to overcome residual piston effects.* Proc. SPIE, 10703 (2018)